Classical dynamics and quantization of billiards with n-fold symmetry

Ricardo Batista do Carmo[1], Tiago Araújo Lima[2]

[1]Instituto Federal de Alagoas, Piranhas, Brasil, ricardo.carmo@ifal.edu.br
[2]Universidade Federal Rural de Pernambuco, Recife, Brasil, tiago.araujol@ufrpe.br

Over the course of our recent research, we have numerically investigated both the classical dynamics and the quantization of billiards possessing only discrete rotational symmetry C_n. For polygonal boundaries, we analyzed ergodicity and the degree of mixing exhibited by trajectories in these systems [1]. In the quantization of hexagonal billiards, we observed that the spectral statistics are well described by the Gaussian Orthogonal Ensemble (GOE) for singlet states and by the Gaussian Unitary Ensemble (GUE) for doublet states [2], distributions historically associated with chaotic systems, as first proposed in 1996 for a billiard with C_3 symmetry [3].

For a family of chaotic billiards with smooth boundaries and C_n symmetry, the classical dynamics reveal that certain subregions of phase space are visited more frequently by chaotic trajectories, a phenomenon known as stickiness [4]. We investigate the relationship between the fraction of sticky regions in classical phase space and the deviations of the energy subspectra from GOE and GUE statistics. Our results suggest the existence of correlations between the energy distributions associated with different symmetry subspaces. In addition, we discuss aspects related to the superposition of the different energy subspectra.

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