LIMIT CYCLES ON HYBRID SYSTEMS

Jaume Llibre[1], **Angela C. T. Sánchez**[2], **Durval J. Tonon**[2]

[2] Departament de Matemàtiques, Universitat Autònoma de Barcelona, Barcelona, Spain

[1] Institute of Mathematics and Statistics, University Federal of Goiás, Goiania, Goiás, Brazil

In this work, we study the maximum number of limit cycles in some classes of discontinuous piecewise hybrid differential systems formed by two rigid systems separated by a straight line. These rigid systems are formed by a linear center plus a homogeneous polynomial of degree 2, 3, 4, 5, 6. Interest in studying the limit cycles of discontinous piecewise hybrid differential systems has increased in recent years due to the wide range of applications of these systems and their connection to the classical *Hilbert's 16th problem*, which asks whether there exists an upper bound for the number of limit cycles in planar polynomial vector fields of a given degree. At present, this problem remains open.

Our main objective is to determine the maximum number of limit cycles that can arise in these systems under study; that is, to solve the extended version of *Hilbert's 16th problem* for this class of discontinuous piecewise differential systems.

Thus, our main result states that the maximum number of limit cycles for this class of systems is at most one when $m = n$, and this bound is achieved. For $m = 2$ and $n = 3, 4, 5, 6$, the maxima are 2, 3, 4, and 5, respectively. For $m = 3$ and $n = 4, 5, 6$, the corresponding bounds are 4, 2. In the case $m = 4$, the maximum is 6 for $n = 5$ and 10 for $n = 6$. Finally, when $m = 5$ and $n = 6$, the system admits at most 8 limit cycles.

In this way, we solve the extended version of *Hilbert's 16th problem* for a class of discontinuous piecewise hybrid differential systems separared by the straight line $y = 0$ and formed by two rigid polynomial differential systems.

References

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