# Numerical evidence of monotonicity for the generalized 2D Zakharov-Kusnetzov equation

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In this work, we consider the initial value problem (IVP) for the 2D generalized Zakharov-Kuznetsov (gZK) equation:

$$\begin{cases} u_t + \partial_x \Delta u \pm \partial_x(u^{k+1}) = 0, & (x,y) \in \mathbb{R}^2, \quad t > 0, \quad k \in \mathbb{N} \\ u(x,y,0) = u_0(x,y). \end{cases} \tag{gZK}$$

The gZK equation is a two-dimensional version to the so-called generalized Korteweg-de Vries (gKdV) equation:

$$\begin{cases} u_t + \partial_x^3 u \pm \partial_x(u^{k+1}) = 0, & x \in \mathbb{R}, \quad t > 0, \quad k \in \mathbb{N} \\ u(x,0) = u_0(x). \end{cases} \tag{gKdV}$$

In the context of nonlinear dispersive equations, *scattering* is the term used to describe the phenomenon where the time evolution of certain initial data is well approximated by the solution of a linear equation. In this context, the nonlinearity effects are not capable of producing, for example, *solitary wave* solutions or the collapse of the wave, known as the *blow-up* phenomenon.

The gKdV equation is a widely known nonlinear dispersive equation and has been deeply studied since its formulation (see, for example, [11, 2, 3, 7]). For that reason, analytical, conclusive results regarding the occurrence of scattering based on properties of the solutions are available, such as *Tao's monotonicity formula* [16], for example. In general, said estimate ensures that the *center of mass* and *center of energy* of a solution to the *defocusig* gKdV (with the negative sign on the nonlinear term) must move with different speeds. This allows us to preclude soliton-like solutions and is a crucial step in proving the occurrence of scattering.

The gZK equation, in turn, has been receiving considerable attention recently [1, 5, 6, 8, 9, 12, 14, 4, 15, 10, 13]. Since the gZK equation is essentially a multidimensional version of the gKdV equation, it is natural to expect that such results could be proved, with the necessary modifications, for the gZK equation. However, because of the inclusion of extra dimensions, the analogous argument used to prove the *monotonicity formula* is more delicate. Therefore, it is an open question whether an analogous inequality can be achieved.

Consequently, in this work, we use numerical tools to solve the IVP associated with the defocusing gZK equation, with different classes of initial data, to investigate the behavior of the *center of mass* and *center of energy* of the solutions obtained. With this, we found considerable numerical evidence that an analogous *monotonicity formula* can be obtained for the gZK equation.

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## Referências


[1] H. A. Biagioni and F. Linares. Well-posedness results for the modified zakharov-kuznetsov equation. *Nonlinear Equations: Methods, Models and Applications*, pages 181–189, 2003.

[2] B. Birnir, C. E. Kenig, G. Ponce, N. Svanstedt, and L. Vega. On the ill-posedness of the ivp for the generalized korteweg-de vries and nonlinear schrödinger equations. *Journal of the London Mathematical Society*, (53):551–559, 1996.

[3] J. Colliander, G. Keel, M andStaffilani, H. Takaoka, and T. Tao. Sharp global well-posedness for kdv and modified kdv on r and t, j. *Journal of the American Mathematical Society*, (16), 2001.

[4] S. a. Correia and S. Kinoshita. Global well-posedness and scattering for the 2D modified Zakharov-Kuznetsov equation. *J. Math. Pures Appl. (9)*, 211:Paper No. 103884, 25, 2026.

[5] A. V. Famisnkii. The cauchy problem for the zakharov–kuznetsov equation. *Differ. Uravn.*, (31):1070–1081, 1995.

[6] L. Farah, F. Linares, and A. Pastor. A note on the 2d generalized zakharov-kuznetsov equation: local, global and scattering results. *Journal of Differential Equations*, (253):2558–2571, 2012.

[7] L. Farah, F. Linares, A. Pastor, and N. Visciglia. Large data scattering for the defocusing supercritical generalized kdv equation. *Communications in Partial Differential Equations*, (43):118–157, 2018.

[8] E. Hussain, S. Malik, A. Yadav, S. A. A. Shah, M. A. B. Iqbal, A. E. Ragab, and H. M. A. Mahmoud. Qualitative analysis and soliton solutions of nonlinear extended quantum zakharov-kuznetsov equation. *Nonlinear Dynamics*, (112):19295–19310, 2024.

[9] S. Islam, M. N. Alam, M. F. Al-Asad, and C. Tunç. An analytical technique for solving new computational solutions of the modified zakharov-kuznetsov equation arising in electrical engineering. *Journal of Applied and Computational Mechanics*, (7):715–726, 2021.

[10] C. E. Kenig, N. Pavlović, G. Staffilani, and L. Velasco. Complex-valued modified zakharov kuznetsov equation, 2026.

[11] C. E. Kenig, G. Ponce, and L. Vega. Well-posedness and scattering results for the generalized Korteweg-de Vries equation via the contraction principle. *Comm. Pure Appl. Math.*, 46(4):527–620, 1993.

[12] C. Klein, S. Roudenko, and N. Stoilov. Numerical study of Zakharov-Kuznetsov equations in two dimensions. *Journal of Nonlinear Science*, 31:Paper No. 26, 28, 2021.

[13] C. Klein, S. Roudenko, and N. Stoilov. The 3d critical zakharov–kuznetsov equation: blow-up and soliton dynamics, 2026.

[14] S. Raut, S. Roy, R. R. Kairi, and P. Chatterjee. Approximate analytical solutions of generalized zakharov–kuznetsov and generalized modified zakharov–kuznetsov equations. *International Journal of Applied and Computational Mathematics*, 7:1–25, 2021.

[15] S. Roudenko and J. Holmer. Spectral property for the 2d zakharov–kuznetsov equation. *Theoretical and Mathematical Physics*, 226(3):404–422, Mar. 2026.

[16] T. Tao. Two remarks on the generalised korteweg de-vries equation. *Discrete and Continuous Dynamical Systems*, (18):1–14, 2007.