# Galilean and Scaling Symmetries in the Characterization of Line Soliton Phases

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**Abstract**

This research focuses on the analysis and characterization of line soliton (oblique) solutions of the Kadomtsev–Petviashvili equation (KP-II). In particular, a relationship is established between the soliton of the Korteweg–de Vries equation (KdV), reinterpreted in the KP-II framework, and the line soliton as a specific solution of the KP-II equation. This approach allowed the characterization of the phase of the KP-II line soliton solution through the use of Airy-type, Heat-type, and Wronskian operators, investigating their interactions with the profile and phase of these solutions. To achieve this, it was essential to introduce a mixed operator, defined as a specific composition of a Galilean transformation and a scaling.

Recent results concerning the specific properties of the profiles and phases of these solutions were employed [2]. In this setting, the parameters $\omega$ and $\beta$ of the mixed operator were precisely determined; these parameters allow a KP-II line soliton to be transformed into a KdV soliton. A key and particularly challenging aspect was the description of the inverse mixed operator in order to establish the link between solutions.

Crucially, this characterization is expressed directly in terms of the Galilean and scaling symmetries of KP-II, whereas the previous characterization in [2] required translating the operator conditions into cones, our formulation reinterprets the Airy- and Heat-type conditions through these symmetries and preserves the Wronskian condition as $\Omega\,\Omega_{xxxx} - (\Omega_{xx})^2 = 0$, thus characterizing the phase without the cone structure of [2]. Furthermore, equations replicating the behavior of the aforementioned operators were derived, highlighting their importance in the deeper understanding of these solutions and the characterization of their phases.

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