Presymplectic geometry of singular Hamiltonian systems: the Faddeev-Jackiw method

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In Lagrangian and Hamiltonian dynamics, a system is said to be singular if, due to some degeneracy in the Lagrangian function, one cannot obtain all Lagrangian or Hamiltonian equations for each generalized coordinate by usual methods such as direct application of variational calculus. In particular, the treatment of so-called singular systems is an important subset of Hamiltonian dynamics, both from the point of view of their mathematical properties, but also due to their importance for quantum physics. In 1950, Dirac proposed a method for obtaining Hamiltonian equations for singular systems by building a generalized Poisson bracket algebra structure [3]. In 1988, L. Faddeev and R. Jackiw proposed a new method [4], which was later augmented by Barcelos-Neto and Wotsazek [1]. This method relies more directly on the theory of presymplectic geometry, as we cannot directly apply the regular symplectic theory to singular systems. The central point is a generalized Darboux theorem for presymplectic forms, which can be constructed using Frobenius' theorem combined with parametric versions of familiar results such as Poincare's lemma [2, 5]. This theory, in the end, allows us to obtain Hamiltonian equations for singular systems, both those with kinematic constraints, as well as those with gauge conditions.

References

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