A Lins Neto's family of foliations and elliptic fibrations and its metamorphoses on Rational, Kummer and Abelian surfaces

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Along three works, A. Lins Neto studied one-parameter families of holomorphic singular foliations having special features: a) the singularities of the foliations have the same non-degenerate analytical types for all parameters out of a finite set; b) the Seidenberg reduction of singularities of all foliations is done by means of a fixed set of blowing ups; c) for generic parameters, the Seidenberg-reduced foliations are turbulent foliations relatively to all elliptic fibrations present in the family, which arise for an infinite, countable, not discrete subset of parameters. He proved that, on the complex projective plane, there are four families of foliations with the properties a), b), c), up to biholomorphism. Here we focus on the fourth family. We describe linear and Cremona symmetries of the family, used to effectively produce infinite number of elliptic pencils in the family. In order to obtain a formula for the degrees of the generic curves of all elliptic pencils, we were led to consider liftings of the fourth family and its symmetries to Kummer and Abelian surfaces. Moreover: we present finite quotients of the family, either by a Cremona symmetry of the plane or by a Van Geemen-Sarti involution of a Kummer surface; we show the birational relation with a S. Kondo's model for a K3 surface; and we show relations between the fourth family and commuting polynomial vector fields.