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On the ZK Equation with a Low-Regularity Wavemaker

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(joint work with Gleb G. Doronin)

Abstract

The Zakharov–Kuznetsov (ZK) equation [4], is known to be locally well-posed in $H^s(\mathbb{R}^2)$ for $s > -1/4$, and globally well-posed in $L^2(\mathbb{R}^2)$, [3]. Real flows in channels-like domains, however, require some boundary conditions to be imposed. These conditions provide both technical difficulties to construct solutions and qualitative features of solutions obtained, [2]. Our aim is to study boundary configuration that improves regularity of ZK solutions when they behave like solutions for the KdV equation.

We establish the local well-posedness in L^2 for the two-dimensional ZK posed on a half-strip with a non-homogeneous boundary condition (wavemaker), and homogeneous Dirichlet conditions along the transverse boundaries. Our approach is based on Bourgain-type spaces adapted to the ZK dispersive structure, combined with anisotropic smoothing and boundary trace estimates. Technically, the boundary wavemaker is incorporated as a forcing term, see [1], in the corresponding Cauchy problem on the whole $\mathbb{R}^2$. This formulation does not affect the resonance structure, and thereby preserves the regularity thresholds. Our approach provides a base for future negative regularity analysis as well as for possible gZK consolidation.

References

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