

Poster Proposal

Well-Posedness and Solitary Waves in Higher-Order KdV-Type Equations

Chandler Haight¹, Svetlana Roudenko¹, Beckett Sanchez², and Diana Son¹

¹Department of Mathematics & Statistics Florida International University, Miami, FL, USA

²Department of Mathematics, Brown University, Providence, RI, USA

Abstract

We study the fifth-order generalized Korteweg–de Vries equation

$$\partial_t u - \partial_x^5 u - 2a \partial_x^3 u + \gamma |u|^\alpha \partial_x u = 0, \quad a, \gamma \in \mathbb{R}, \quad (1)$$

with particular interest in the low-power regime $0 < \alpha < 1$.

We also consider a broader class of higher-order KdV-type equations,

$$\partial_t u - \partial_x^5 u - 2a \partial_x^3 u + \sum_{k=1}^N c_k |u|^{\alpha_k} u^{n_k} |\partial_x u|^{\beta_k} \partial_x u (|\partial_x^2 u|^{\gamma_k} \partial_x^2 u)^{p_k} = 0, \quad (2)$$

where

$$c_k \in \mathbb{R}, \quad \alpha_k, \beta_k, \gamma_k \geq 0, \quad n_k, p_k \in \mathbb{N} \cup \{0\}.$$

For the low-power case $0 < \alpha < 1$, the main difficulty is that $|u|^\alpha$ is not locally Lipschitz at the origin. We therefore work with slowly decaying initial data satisfying a weighted non-vanishing condition and prove local well-posedness in the weighted space $\mathcal{X}$. The fifth-order dispersion also gives additional smoothing, which allows us to treat a more general class of nonlinearities involving derivatives up to second order.

We also study the equation numerically. First, we compute solitary-wave profiles from the corresponding fourth-order stationary equation. Depending on the value of the lower-order dispersion parameter a , these profiles may be positive or sign-changing. We then evolve initial data of the form

$$u_0(x) = \frac{A}{1 + |x|^2},$$

which is in the weighted setting used in the well-posedness result. We vary A , a , and α and follow the resulting solution in time. In several cases the initial bump sheds radiation and leaves behind a coherent traveling structure. We compare these evolved structures with the computed ground-state profiles, including the sign-changing cases.