The well-posedness theory for the quadratic Schrödinger system in 1D

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We present sharp local well-posedness (LWP) results, depending on the ratio dispersion ratio $a \in \mathbb{R} \setminus \{0\}$, for the initial-value problem associated with the quadratic nonlinear Schrödinger system:

$$\begin{cases} iu_t + u_{xx} + \beta u = \theta \bar{u} v, \\ iv_t + av_{xx} - \lambda v = \gamma u^2, \end{cases} \qquad (t, x) \in \mathbb{R} \times \mathbb{R},$$

with initial data $(u_0, v_0) \in H^k(\mathbb{R}) \times H^s(\mathbb{R})$. The proof combines standard fixed-point arguments for the near-diagonal case, normal form reductions paired with nonlinear smoothing for the far-off diagonal region, and a gauge transform method for the low-regularity regime. By identifying the precise LWP regularity range, we establish optimality by constructing counterexamples to show ill-posedness in the complement of the LWP region. Furthermore, we extend the known global well-posedness theory to the off-diagonal regime. This significantly generalizes the well-posedness theory previously established in the literature.