# Orbit-equivalences for two-step Riemannian nilmanifolds

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We say that two left invariant metrics in a nilmanifold are orbit-equivalent if there exists a homeomorphism between their respective unit tangent bundles that intertwines their geodesic flows but is not time-preserving. It is proven in [GM98] that there exist large classes of compact two-step Riemannian nilmanifolds where geodesic conjugacy between two left invariant metrics, i.e. a time-preserving orbit-equivalence between their geodesic flows, implies that both metrics must be isometric. In this work, we generalize some of these ideas to general orbit-equivalences for some classes of compact two-step Riemannian nilmanifolds. As a consequence, we obtain some results that suggest a rigid geometric and dynamical behaviour, in contrast to their topological nature. We also obtain examples of left invariant metrics that are not orbit-equivalent.

## References

[GM98] C. Gordon and Y. Mao, Geodesic conjugacies of two-step nilmanifolds., *Michigan Math. J.* **45** no. 1 (1998), 451–481.